

# PICOF 2018 Conference

on

## Inverse Problems, Control and Shape Optimization

18 - 19 - 20 June 2018

Co-organized by

DEPARTMENT OF MATHEMATICS, FACULTY OF ARTS AND  
SCIENCES

and

CENTER FOR ADVANCED MATHEMATICAL SCIENCES  
at the

AMERICAN UNIVERSITY OF BEIRUT, LEBANON



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## Conference Venue

Auditorium B1, College Hall,  
American University of Beirut,  
Bliss Street, Beirut, Lebanon

## Conference Dinner

Monday 18 June 2018, 7:30pm  
Faouzi Burj Al Hamam Restaurant,  
Golden Tulip Serenada Hotel, Hamra  
45\$, Payment at Registration Desk  
before 18 June at 2:30pm

## Conference Excursion

Tuesday 19 June 2018, 1:15pm  
Jeita, Harissa and Byblos  
45\$, Payment at Registration Desk  
before 19 June at 11am  
[www.cs.aub.edu.lb/picof2018/index.html](http://www.cs.aub.edu.lb/picof2018/index.html)

## Conference Sponsors

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Informatique et en Automatique (INRIA)



# 1 What is PICOFF?

**PICOFF (Problèmes Inverses, Contrôle et Optimisation de Formes)** is a bi-annual international scientific conference which assembles senior researchers and scientists as well as a number of young researchers dealing with mathematical and numerical problems in the fields of inverse problems, control and shape optimization. It aims to gather the community of researchers in academia and industry working in these three fields.

## 2 PICOFF 2018 Committees

### Coordination

Amel Ben Abda  
*Tunis El Manar University*  
Nabil Nassif  
*American University of Beirut*

### Local Organizing Committee

Sophie Moufawad  
*American University of Beirut*  
Toufic El Arwadi  
*Beirut Arab University*  
Gihane Mansour  
*Saint Joseph University*  
Wafic Sabra  
*American University of Beirut*  
Hassan Saoud  
*Lebanese University*  
Ali Wehbe  
*Lebanese University*

### Scientific Committee

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*Ecole Mohammadia of Rabat*  
Gregoire Allaire  
*Ecole Polytechnique*  
Hajer Bahouri  
*University of Paris-Est - Créteil*  
Elena Beretta  
*Politecnico di Milano*

Faker, Ben Belgacem  
*University of Technology Compiègne*  
Mourad Bellassoued  
*Tunis El Manar University*  
Jacques Blum  
*University of Nice*  
Housseem Haddar  
*Ecole Polytechnique*  
Leila Issa  
*Lebanese American University*  
Moez Khenissi  
*University of Sousse*  
Juliette Leblond  
*INRIA, Sophia-Antipolis*  
Daniel Lesnic  
*University of Leeds*  
Mohamed Masmoudi  
*University of Paul Sabatier*  
Nabil Nassif  
*American University of Beirut*  
Toni Sayah  
*Saint Joseph University*  
Alexandru Tamasan  
*University of Central Florida*  
Ouahiba Zair  
*University of Science and Technology  
Houari Boumediene*  
Enrique Zuazua  
*Universidad Autónoma de Madrid*

### 3 Program-at-a-glance

#### Monday, June 18

09:00am - 09:30am	Opening Ceremony <sup>1</sup>	<i>Auditorium B1</i>
09:30am - 10:20am	<b>IP1</b> Samuel Amstutz (University of Avignon)	<i>Auditorium B1</i>
10:20am - 11:10am	<b>IP2</b> Pierre Lissy (University of Paris Dauphine)	<i>Auditorium B1</i>
11:10am - 11:30am	Coffee Break	<i>Auditorium B1</i>
11:30am - 01:00pm	<b>Concurrent Sessions</b>	
	<b>MS1</b> (Organizer: Ahmed El Hajj)	<i>Auditorium B1</i>
	Contributed Talks	<i>CR<sup>2</sup>, 4th floor</i>
01:00pm - 02:10pm	Lunch Break	<i>Auditorium B1</i>
02:10pm - 03:00pm	<b>IP3</b> Faouzi Triki (University of Grenoble-Alpes)	<i>Auditorium B1</i>
03:00pm - 05:00pm	<b>MS2</b> (Organizer: Sidi-Mahmoud Kaber)	<i>Auditorium B1</i>
07:30pm -	Conference Dinner	

#### Tuesday, June 19

09:00am - 09:50am	<b>IP4</b> Gilles Lebeau (University of Nice Sophia-Antipolis)	<i>Auditorium B1</i>
09:50am - 10:15am	Coffee Break	<i>Auditorium B1</i>
10:15am - 12:45pm	<b>Concurrent Sessions</b>	
	<b>MS3</b> (Organizer: Faouzi Triki)	<i>CR, 4th floor</i>
	<b>MS4</b> (Organizer: Faker Ben Belgacem)	<i>Auditorium B1</i>
	<b>MS5</b> (Organizer: Rajae Aboulaich)	<i>CR, 3rd floor</i>
12:45pm - 01:15pm	Lunch Break	<i>Auditorium B1</i>
01:15pm -	Cultural Excursion	

#### Wednesday, June 20

09:00am - 09:50am	<b>IP5</b> Belhassen Dehman (LAMSIN, University of Tunis El Manar)	<i>Auditorium B1</i>
09:50am - 10:40am	<b>IP6</b> Lassaad Aloui (University of Tunis El Manar)	<i>Auditorium B1</i>
10:40am - 11:00am	Coffee Break	<i>Auditorium B1</i>
11:00am - 11:50am	<b>IP7</b> Hend Ben Ameer (LAMSIN, University of Tunis El Manar)	<i>Auditorium B1</i>
11:50am - 01:00pm	Lunch Break	<i>Auditorium B1</i>
01:00pm - 03:00pm	<b>Concurrent Sessions</b>	
	<b>MS6</b> (Organizer: Moez Khenissi)	<i>CR, 3rd floor</i>
	<b>MS7</b> (Organizer: Zakaria Belhachmi)	<i>Auditorium B1</i>
	<b>MS8</b> (Organizer: Nejib Zemzemi)	<i>CR, 4th floor</i>
03:00pm - 03:15pm	Closing Ceremony	<i>Auditorium B1</i>

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<sup>1</sup>Registration is open and Coffee is served

<sup>2</sup>CR: Conference Room

## 4 Invited Plenary Speakers

All Invited Plenary Presentations will take place in Auditorium B1, College Hall.

**Monday, June 18, 9:30am - 10:20am**

**IP1 A consistent relaxation of optimal design problems for coupling shape and topological derivatives**

*Samuel Amstutz, University of Avignon*

**Monday, June 18, 10:20am - 11:10am**

**IP2 Controllability of coupled parabolic systems**

*Pierre Lissy, University of Paris Dauphine*

**Monday, June 18, 2:10pm - 3:00pm**

**IP3 Identification of an inclusion in Multifrequency Electrical Impedance Tomography**

*Faouzi Triki, University of Grenoble-Alpes*

**Tuesday, June 19, 9:00am - 9:50am**

**IP4 Spectral inequalities for Schrödinger operators**

*Gilles Lebeau, University of Nice Sophia-Antipolis*

**Wednesday, June 20, 9:00am - 9:50am**

**IP5 Observability for the wave equation with rough coefficients**

*Belhassen Dehman, LAMSIN, University of Tunis El Manar*

**Wednesday, June 20, 9:50am - 10:40am**

**IP6 Indirect stabilization of coupled wave-type systems on exterior domains**

*Lassaad Aloui, University of Tunis El Manar*

**Wednesday, June 20, 11:00am - 11:50am**

**IP7 Some inverse problems in hydrogeology**

*Hend Ben Ameer, LAMSIN, University of Tunis El Manar*

## 5 Program Schedule

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### Monday, June 18

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#### Opening Ceremony

09:00am - 09:30am

AUDITORIUM B1

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#### IP1

09:30am - 10:20am

AUDITORIUM B1

#### **A consistent relaxation of optimal design problems for coupling shape and topological derivatives**

SAMUEL AMSTUTZ, UNIVERSITY OF AVIGNON

.....

#### IP2

10:20am - 11:10am

AUDITORIUM B1

#### **Controllability of coupled parabolic systems**

PIERRE LISSY, UNIVERSITY OF PARIS DAUPHINE

.....

#### MS1

#### **Inverse Source Problems**

11:30am - 01:00pm

AUDITORIUM B1

Inverse problems are of increasing importance in several applied and practical domains. Beside their applications to natural phenomena such as, pollution in the environment, antenna synthesis problem and discrete dislocation problems, these inverse problems have been widely used in several biomedical imaging techniques as the photo- and thermo-acoustic tomography. In addition to that, electroencephalography/magnetoencephalography (EEG/MEG) problems and bioluminescence tomography (BLT) form two important non-invasive biomedical imaging methods that motivate inverse source problems. The aim of EEG/MEG problems, used in the epilepsy disease treatment, is to obtain a fairly accurate localization of the epileptogenic sources using electrical and magnetic measures on the scalp. While BLT consists in determining an internal bioluminescent source distribution generated by luciferase induced by reporter genes from external optical measurements. We present in this minisymposium, some stability results as well as identification methods that allow precise localization of these foci and their distributions. More precisely, we show a sensitivity result for the EEG model in neonates which takes into account the presence and ossification process of fontanels via the conductivity equation. Then, we show a stability result, for BLT model and antenna synthesis problem, considering the multiple frequencies case via Helmholtz equation.

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*Organizers: Ahmed El Hajj, University of Technology Compiègne,  
Hayat Moustafa, Lebanese University*

#### **11:30 - 12:00 Stability estimates for an inverse source problem in seawater intrusion model**

*Hayat Moustafa, Lebanese University*

#### **12:00 - 12:30 EEG source localization in neonates**

*Stephanie Lohrengel, Reims-Champagne Ardenne University*

#### **12:30 - 01:00 On the inverse conductivity problem with a single internal measurement**

*Faouzi Triki, University of Grenoble - Alpes*

## Contributed Talks

11:30am - 01:00pm

CONFERENCE ROOM 4TH FLOOR

### 11:30 - 12:00 Optimal Control with Exit Time

*Samer Dweik*, Paris-Sud University

### 12:00 - 12:30 A Regularized D-Bar Algorithm for the 2D Real Inverse Conductivity Problem

*Toufic El Arwadi*, Beirut Arab University

### 12:30 - 01:00 Stochastic homogenization of a front propagation problem with unbounded velocity

*Ahmed Hajej*, Pantheon-Assas University

## Lunch Break

01:00pm - 02:10pm

AUDITORIUM B1

## IP3

02:10pm - 03:00pm

AUDITORIUM B1

### Identification of an inclusion in Multifrequency Electrical Impedance Tomography

FAOUZI TRIKI, UNIVERSITY OF GRENOBLE-ALPES

## MS2

### Control of some wave and transport models

03:00pm - 05:00pm

AUDITORIUM B1

This minisymposium bring together young researchers working on control problems. They will present recent developments on controlability and observability of propagating models. Both theo-

retical and numerical aspects will be addressed.

*Organizers: Sidi-Mahmoud Kaber*, University of Paris 6 (UPMC), *Ali Wehbe*, Lebanese University

### 03:00 - 03:30 Problemes de controle lies aux mouvements de foules

*Michel Duprez*, Marseille University

### 03:30 - 04:00 A spectral approach to the indirect boundary control of the Timoshenko system

*Mouhammad Ghader*, Lebanese University and Paris Sud University

### 04:00 - 04:30 Internal controls for a semi-discrete problem with fractional Laplacian using finite-difference method

*Pierre Lissy*, University of Paris Dauphine

### 04:30 - 05:00 Indirect stability of the wave equation with a dynamic boundary control

*Mohammad Sammoury*, Lebanese International University and Al Maaref University

## Conference Dinner

07:30pm -

Faouzi Burj El Hamam

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## Tuesday, June 19

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## IP4

09:00am - 09:50am

AUDITORIUM B1

### Spectral inequalities for Schrödinger operators

GILLES LEBEAU, UNIVERSITY OF NICE SOPHIA-ANTIPOLIS

.....

### **Coffee Break**

09:50am - 10:15am

AUDITORIUM B1

.....

### **MS3**

#### **Inverse Scattering Problems**

10:15am - 12:45pm

CONFERENCE ROOM, 4TH FLOOR

Inverse scattering problems are important in diverse scientific areas such as radar and sonar, geophysical exploration, medical imaging, near-field optical microscopy, and nano-optics. Due to the complexity of material properties and uncertainty in physical models and parameters, precise modeling and accurate computing present challenging and significant mathematical and computational questions, and remain the subject matter of much ongoing research. This minisymposium seeks to bring together leading researchers in various fields that involve inverse scattering theory to present recent developments, promote exchange of ideas, and discuss new directions including treatment of internal and boundary data. The talks will cover precisely identification of physical coefficients in waveguides, fluids and elastic media.

*Organizer: Faouzi Triki, University of Grenoble-Alpes*

#### **10:15 - 10:45 Manipulating particles in a fluid with standing acoustic waves**

*Fernando Guevara Vasquez, University of Utah*

#### **10:45 - 11:15 Direct linear inversion for discontinuous elastic parameters recovery from internal displacements**

*Laurent Seppecher, Ecole Centrale Lyon*

#### **11:15 - 11:45 Stability in inverse problems for the magnetic Schrodinger equation**

*Eric Soccorsi, University of Aix-Marseille*

#### **11:45 - 12:15 On the inverse problem of electro-seismicity in porous media**

*Faouzi Triki, University of Grenoble-Alpes*

#### **12:15 - 12:45 Around the Calderón problem in a waveguide**

*Yavar Kian, University of Aix-Marseille*

.....

### **MS4**

#### **Inverse Scattering Problems**

10:15am - 12:15pm

AUDITORIUM B1

*Organizer: Faker Ben Belgacem, University of Technology Compiègne*

#### **10:15 - 10:45 Lavrentiev-Finite Element Methods for Data Completion Problems with Domain Extensions**

*Faker Ben Belgacem, University of Technology Compiègne*

#### **10:45 - 11:15 Study of inverse problems by optimization methods**

*Fabian Caubet, Toulouse University*

#### **11:15 - 11:45 Stecklov-Poincar algorithm for the Cauchy problem : regularization by spectral analysis**

*Kadri Mohamed Larbi, LAMSIN, University of Tunis El Manar*

#### **11:45 - 12:15 Shape Sensitivity Analysis in Linear Elasticity**

*Bochra Mejri, University of Tunis El Manar*

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**MS5  
Biomath**

10:45am - 12:45pm  
CONFERENCE ROOM, 3RD FLOOR

*Organizer: Rajae Aboulaich, Ecole Mohammadia of Rabat*

**10:45 - 11:15 A 3D multiscale modelling framework to investigate the effects of acquired mutations in the EGFR signaling on tumor progression**  
*Anass Bouchnita, Uppsala University*

**11:15 - 11:45 Simulation of blood flow in a stenosed artery and fractional flow reserve computation**  
*Keltoum Chahour, University of Nice Sophia-Antipolis*

**11:45 - 12:15 Cell plasticity and drug resistance in cancer, with perspectives in optimal therapeutic control**  
*Jean Clairambault, INRIA*

**12:15 - 12:45 Helfrich and differential area modeling of vesicles**  
*Olivier Pantz, LJAD, University of Nice Sophia-Antipolis*

**Lunch Break, then Cultural Excursion**  
12:45pm - 01:15pm  
AUDITORIUM B1

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**Wednesday, June 20**

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**IP5**  
09:00am - 09:50am  
AUDITORIUM B1

**Observability for the wave equation with rough coefficients**  
BELHASSEN DEHMAN, LAMSIN, UNIVERSITY OF TUNIS EL MANAR

.....  
**IP6**  
09:50am - 10:40am  
AUDITORIUM B1  
**Indirect stabilization of coupled wave-type systems on exterior domains**  
LASSAAD ALOUI, UNIVERSITY OF TUNIS EL MANAR

.....  
**Coffee Break**  
10:40am - 11:00am  
AUDITORIUM B1

.....  
**IP7**  
11:00am - 11:50am  
AUDITORIUM B1  
**Some inverse problems in hydrogeology**  
HEND BEN AMEUR, LAMSIN, UNIVERSITY OF TUNIS EL MANAR

.....  
**Lunch Break**  
11:50pm - 01:00pm  
AUDITORIUM B1

.....  
**MS6**  
**Control and Stabilization Problems**  
01:00pm - 02:30pm  
CONFERENCE ROOM, 3RD FLOOR

*Organizer: Moez Khenissi, University of Sousse*

**01:00 - 01:30 Diffusive phenomenon for the**

**wave equation with damping effective at infinity**

*Julien Royer, Paul Sabatier University*

**01:30 - 02:00 Stabilization of coupled wave equation under geometric conditions**

*Ali Wehbe, Lebanese University*

**02:00 - 02:30 Stabilization of multidimensional wave equation with locally boundary fractional dissipation law under geometric conditions**

*Mohammad Akil, Lebanese University*

## **MS7**

*01:00pm - 03:00pm*

AUDITORIUM B1

*Organizer: Zakaria Belhachmi, LMIA Haute Alsace*

**01:00 - 01:30 Detection of inclusions while recovering boundary data in Stokes Flows using Nash strategies**

*Abderrahmane Habbal, University of Nice Sophia-Antipolis*

**01:30 - 02:00 Optimization of current carrying multi cables using topological and shape gradient strategy**

*Meftahi Belhassen, LAMSIN, University of Tunis El Manar*

**02:00 - 02:30 Image restoration based on the  $p(x)$ -biharmonic operator**

*Hamdi Houichet, LAMSIN, University of Tunis El Manar*

**02:30 - 03:00 Modeling Fractures as Inter-**

**faces: Model Darcy-Stokes**

*Kawther Dhifaoui, Ecole Nationale d'Ingénieurs de Tunis*

## **MS8**

**Inverse problems in Cardiac electrophysiology**

*01:00pm - 03:00pm*

CONFERENCE ROOM, 4TH FLOOR

This mini-symposium deals with different inverse problems in computational cardiac electrophysiology

*Organizer: Nejib Zemzemi, INRIA Bordeaux Sud-Ouest*

**01:00 - 01:30 Inverse ECG problem via the factorization method of boundary value problems.**

*Jacques Henry, INRIA Bordeaux Sud-Ouest*

**01:30 - 02:00 A game approach to solve the coupled problem of conductivity identification and data completion in cardiac electrophysiology**

*Rabeb Chamekh, LAMSIN, University of Tunis El Manar*

**02:00 - 02:30 On the identification of multiple space dependent ionic parameters using the bidomain system in cardiac electrophysiology modeling**

*Moncef Mahjoub, LAMSIN, University of Tunis El Manar*

**02:30 - 03:00 Homogenization theory for the derivation of diffusion models**

*Marwa Kchaw, LAMSIN, University of Tunis El Manar*

## 6 Abstracts of Invited Plenary Talks

### IP1

#### **A consistent relaxation of optimal design problems for coupling shape and topological derivatives**

I will present a general procedure for approximating a ‘black and white’ shape and topology optimization problem with a density optimization problem, allowing for the presence of ‘grayscale’ regions. The construction relies on a regularizing operator for smearing the characteristic functions involved in the exact optimization problem, and on an interpolation profile, which endows the intermediate density regions with fictitious material properties. Under mild hypotheses on the smoothing operator and on the interpolation profile, we prove that the features of the approximate density optimization problem (material properties, objective function, etc.) converge to their exact counterparts as the smoothing parameter vanishes. In particular, the Fréchet derivative of the approximate objective functional with respect to the density function converges to either the shape or the topological derivative of the exact objective, depending on whether it is evaluated at the boundary of the domain or in its interior. These results shed new light on the connections between these two different notions of sensitivities for functions of the domain and on the construction of consistent interpolation schemes. Related algorithms, including level-set formulations and the incorporation of perimeter penalization, will be discussed and illustrated by numerical outputs.

Samuel Amstutz, University of Avignon

C. Dapogny, LJK, Université Grenoble-Alpes

A. Ferrer, CIMNE, Universitat Polytechnica de Catalunya

### IP2

#### **Controllability of coupled parabolic systems**

Many models of interest involve controlled linear or semi-linear parabolic coupled equations of convection-reaction-diffusion type, for instance in medicine for the treatment of tumors, in chemotaxis, in ecology for the spreading of invasive species, etc. The common point between all these models is that they are underactuated, in the sense that we have less controls than equations in the system, meaning that we can only act directly on some of the (physical, chemical,...) quantities involved in the system. This leads to many mathematical difficulties, notably compared to the scalar parabolic equations. In this talk, we will present some recent internal controllability results on linear toy models, in order to emphasize the crucial role of the structure of the coupling terms and of the control operator. After presenting some results in the case of constant coupling coefficients, we will focus on coupling terms depending on the time and space variable. In this case, only partial results are known, and many unexpected phenomena may appear, notably geometric conditions on the support of the coupling term and control region, minimal time of controllability and lack of approximate controllability.

Pierre Lissy, University of Paris Dauphine

### IP3

#### **Identification of an inclusion in Multifrequency Electrical Impedance Tomography**

In the talk I will present recent results on multifrequency electrical impedance tomography. The inverse problem consists in identifying a conductivity inclusion inside a homogeneous background medium by injecting one current. I will

use an original spectral decomposition of the solution of the forward conductivity problem to retrieve the Cauchy data corresponding to the extreme case of perfect conductor. Considering results based on the unique continuation, I will then prove the uniqueness of the multifrequency electrical impedance tomography, and obtain rigorous stability estimates. Finally, I will present some numerical results inspired by the developed theoretical approach.

Faouzi Triki, University of Grenoble-Alpes,  
H. Ammari, C-H Chun.

#### **IP4**

##### **Spectral inequalities for Schrödinger operators**

We will first recall what are the “spectral inequalities which yield sharp quantification of the unique continuation of the spectral family associated with Laplace-Beltrami operators, and which are used to prove exact controllability for parabolic operators on compact varieties or bounded domains. Next, we will present the spectral inequality in the unbounded case, i.e. for the Schrödinger operator  $-\Delta_g + V(x)$  in  $\mathbb{R}^d$ ,  $d \geq 1$ , with a metric  $g = Id + r$  and a real analytic potential  $V = V(x)$ , under the assumption that  $r, V$  are analytic symbol of negative degree in the complex strip  $|Im(z)| < a$ , for some  $a > 0$ . In particular, this allows long-range potentials.

Gilles Lebeau, University of Nice Sophia-Antipolis  
Ivan Moyano, Centre for Mathematical Sciences,  
University of Cambridge

#### **IP5**

##### **Observability for the wave equation with rough coefficients**

The property of observability for the wave equation is a central question since its linked to

controllability. It has been intensively studied, mainly in a smooth framework ( smooth metric and geometry ). In this lecture, we will present some results on observability/control for the wave equation with rough coefficients.

Belhassen Dehman, LAMSIN, University of Tunis El Manar

#### **IP6**

##### **Indirect stabilization of coupled wave-type systems on exterior domains**

We study the indirect stabilization of systems of two wave-type equations coupled by a localized zero or one order term on exterior domains. Only one of the two equations is directly damped by a localized damping term. We prove that the energy of smooth solutions of the system decays uniformly under geometric conditions on both the coupling and the damping region.

Lassaad Aloui, University of Tunis El Manar

#### **IP7**

##### **Some inverse problems in hydrogeology**

For many hydrogeological problems, such as management and protection of water resources, we need to study the processes of subsurface flow and of transfer of solutes in the subsurface. Since models become more and more complicated and quantitative answers must be given, numerical modeling become more and more sophisticated and mathematicians must also be involved.

We are interested in some inverse problems in porous media: parameter estimation, fracture identification and wells location. All these problems are formulated as optimization problems. The main and common tool in the developed techniques is “ the gradient ” of a convenient function.

An adaptive parameterization algorithm is developed, implemented and applied for the estima-

tion of scalar and vector parameters in porous media. Values of parameters and shapes of hydrogeological zones are unknown. The main tool in the adaptive parameterization approach is a *refinement indicator*: Once the identification problem is set as a minimization of an objective function, the question is what is the effect on this function of allowing discontinuity of the parameter in some candidate location? Refinement indicators give the answer to this question. We combine a posteriori error estimator techniques with adaptive parameterization to improve the resolution of the parameter estimation problem.

While fractures are characterized by discontinuities, the idea is to extend previous indicators to locate fractures. We define *fracture indicators*

and we proceed in an iterative way in order to identify fractures in porous media.

The topological sensitivity analysis method has been recognized as a promising tool to solve topology optimization problems. It consists to provide an asymptotic expansion of a shape functional with respect to the size of a small hole created inside the domain. We apply this method to determine the location of the wells.

To solve the inverse problem where both parametrization and well's location are unknown, we incorporate the topological gradient approach in the adaptive parametrization algorithm; first results are promising.

Hend Ben Ameer, LAMSIN, University of Tunis El Manar

## 7 Abstracts of Minisymposia Talks

### MS1

#### **Stability estimates for an inverse source problem in seawater intrusion model**

This talk deals with an inverse monopolar source problem for the Poisson equation, from interior measurements, whose motivation lies in the seawater intrusion phenomenon. This latter occurs especially in coastal aquifers when saltwater moves from the ocean into freshwater aquifers leading to contamination of drinking water sources. The inverse problem related to this phenomenon consists in identifying wells from interior measurements. Global logarithmic and Lipschitz stability estimates of locations and intensities for monopolar sources are established. To do that, we make an appropriate choice of a test function allowing to show a stability estimate with respect to boundary conditions and then we use an observability inequality for Laplace equation to control it by the interior measurements.

This reveals, on the one hand, the distance between the sources and on the other hand the gap between the associated measures. Finally some results of numerical identifications of sources are shown.

Hayat Moustafa, Lebanese University

### MS1

#### **EEG source localization in neonates**

Electroencephalography (EEG) is a non-invasive functional brain imaging technique that measures the electrical activity of the brain at electrodes located on the scalp. The measurements provide valuable information about sources that are at the origin of pathological brain activity. In this context, EEG is one of the main diagnostic tests in presurgical evaluation for refractory epilepsy. From the mathematical point of view, EEG source reconstruction is an inverse problem that aims to identify the sources responsible of

electrical brain activity from the knowledge of the measured potential on the scalp. The associated forward problem consists in computing the potential on the scalp for a given source located in the brain. In this talk, we propose a mathematical model for the forward EEG problem in neonates and address theoretical and numerical issues of the corresponding inverse problem. The model is able to take into account the presence and ossification process of fontanels which are modeled by a variable conductivity. The sources of focal brain activity are described by a finite sum of electric dipoles that are characterized, respectively, by their position and their moment. The governing partial differential equation is the stationary diffusion equation with piecewise regular coefficients and a source term that derives from the Dirac delta. The inverse problem is solved by minimizing an appropriate functional, and numerical experiments illustrate the performance of the method. Moreover, we perform a sensitivity analysis of the model with respect to variations of the conductivity and discuss the impact of the presence of fontanels on the EEG inverse problem.

Stephanie Lohrengel, Reims-Champagne Ardenne University

M. Darbas, Malal Diallo, LAMFA, UPJV  
Abdellatif El Badia, LMAC, UTC

### **MS1**

#### **On the inverse conductivity problem with a single internal measurement**

In the talk I will present recent results on recovering the conductivity map from a single internal measurement. This inverse problem originated from multi-wave imaging. The objective is to stabilize and improve the resolution in imaging biological tissues. I will first show a stability estimate of Hlder type with weak assumptions on the conductivity map. Then, I will give a convergence

result for the reconstruction of the conductivity coefficient using discontinuous Galerkin method (DG). Finally, I will present some numerical results on synthetic data to validate the theoretical approach.

Faouzi Triki, University of Grenoble - Alpes

### **MS2**

#### **Problèmes de contrôle liés aux mouvements de foules**

Dans cet exposé, nous étudierons la contrôlabilité d'équations aux dérivées partielles de type transport qui apparaissent dans la modélisation des mouvements de foules. Nous contrôlerons ce système en agissant sur la vitesse des individus dans une région donnée de l'espace. Nous montrerons que sous certaines conditions géométriques, il est possible de contrôler de manière approchée le système à l'aide d'un contrôle régulier. Nous étudierons également la contrôlabilité exacte et le temps minimal pour atteindre la cible. Nous terminerons par quelques simulations numériques.

Michel Duprez, Marseille University

### **MS2**

#### **A spectral approach to the indirect boundary control of the Timoshenko system**

In this work, we study the indirect boundary exact controllability of a one-dimensional Timoshenko System. We consider the cases when the speed waves propagate with equal or different speeds. First, we use non harmonic analysis to establish weak observability inequality, which depends on the ratio of the waves propagation speeds. Next, using the HUM method, we prove that the System is exactly controllable, and that the control time can be small.

Mouhammad Ghader, Lebanese University and Paris Sud University

## MS2

### **Internal controls for a semi-discrete problem with fractional Laplacian using finite-difference method**

We consider a finite-difference semi-discrete scheme for the approximation of internal controls of a one-dimensional evolution problem of hyperbolic type involving the spectral fractional Laplacian. The continuous problems is controllable in arbitrary small time. However, the high frequency numerical spurious oscillations lead to a loss of the uniform (with respect to the mesh-size) controllability property of the semi-discrete model in the natural setting. For all initial data in a natural energy space, if we filter the high frequencies of these initial data in an optimal way, we restore the uniform controllability property in arbitrary small time. The proof is mainly based on the moment method.

Pierre Lissy, University of Paris Dauphine

## MS2

### **Indirect stability of the wave equation with a dynamic boundary control**

In this seminar, we consider a damped wave equation with a dynamic boundary control. First, we show the strong stability by applying a theorem due to Arendt and Batty. Next, we prove that our system is not uniformly stable in general, since it is the case of the unit disk. Hence, we look for a polynomial decay rate for smooth initial data for our system. In a first step, by giving some sufficient conditions on the boundary of our domain and by using the exponential decay of the wave equation with a standard damping, we prove a polynomial decay in  $\frac{1}{t^{0.25}}$  of the energy. In a second step, under appropriated condition on the boundary of our system named by the multiplier geometric control, we establish a polynomial decay in  $\frac{1}{t}$  of the energy. Mohammad Sammoury, Lebanese International University

## MS3

### **Manipulating particles in a fluid with standing acoustic waves**

Consider a collection of small particles within a fluid that is subject to a standing acoustic wave. If the particles are neutrally buoyant and less compressible than the fluid, the particles cluster about the nodes of the wave. We present two methods for finding Helmholtz equation solutions that could trap particles in patterns that are desired. The first method assumes the standing acoustic wave is generated in a reservoir by a few transducers and finds the transducer settings that best reproduce a desired pattern of particles. The second method uses results for approximating functions with Helmholtz equation solutions. This problem has possible applications to 3D printing of materials with configurable inclusion locations.

Fernando Guevara Vasquez, University of Utah

## MS3

### **Direct linear inversion for discontinuous elastic parameters recovery from internal displacements**

In this talk, I will present a study of the invertibility and the corresponding stability for the elastography problem from internal data. In medical imaging, it is possible to track the inner fast displacement field of a living tissue using MRI, Optical Coherence Tomography or Ultrafast Ultrasound Imaging. From this data a major problem is to provide a stable and fast method to recover elastic properties of the biological tissue. The displacement field can be generated either by static or dynamic (in time regime or time harmonic regime) solicitations or even by natural sources (heart beats, breathing,...). Most of the time these external forces are not accurately known. In order to avoid iterative inversion procedure, we propose a direct local and linear approach in looking for the inversion the stiffness-to-force op-

erator. If  $\mathbf{u}(x)$  is the inner displacement field, the associated stiffness-to-force operator is given by  $A_{\mathbf{u}} : \mathbf{C} \mapsto -\nabla \cdot (\mathbf{C} : \nabla^s \mathbf{u})$ .

I will present a general approach to numerically invert this kind of linear operators without neither smoothness hypothesis on the unknown tensor  $\mathbf{C}$ , nor boundary knowledge. I will then discuss the general stability question linked to the closed range property of the linear operator  $A_{\mathbf{u}}$ . In a second time, I will focus on the most useful question that is the shear modulus reconstruction. In this case, I will show that under non restrictive piecewise smoothness hypothesis, the inversion is possible with only one measurement. I will then give corresponding stability results in  $L^2$ .

Laurent Seppecher, Ecole Centrale Lyon

### MS3

#### **Stability in inverse problems for the magnetic Schrödinger equation**

In this talk I will discuss the stability issue in the determination of the magnetic potential entering the Schrödinger equation through a finite number of boundary measurements of the solution. This is based on joint works with X. Huang (Tokyo), Y. Kian (Marseille) and M. Yamamoto (Tokyo).

Eric Soccorsi, University of Aix-Marseille

### MS3

#### **On the inverse problem of electro-seismicity in porous media**

In this talk, we study the coupling phenomenon of electromagnetic and seismic waves in porous media. Partial differential equations governing the coupling phenomenon are composed of Maxwell and Biot equations. We are interested in the electro-seismic inverse problem. After deriving the uniqueness of solutions to the forward problem in time domain, we establish stability estimates for the global inversion. Faouzi Triki, Qi Xue, University of Grenoble-Alpes

### MS3

#### **Around the Calderón problem in a waveguide**

Let  $\Omega$  be an unbounded domain of  $\mathbb{R}^3$  associated with a closed waveguide in the sense that there exists  $\omega$  a bounded domain of  $\mathbb{R}^2$  such that  $\Omega \subset \omega \times \mathbb{R}$ . In this talk, we will consider the inverse problem of determining the magnetic field associated with the magnetic potential  $A \in L^\infty(\Omega)^3$  and the electric potential  $q \in L^\infty(\Omega; \mathbb{C})$  appearing in the magnetic Schrödinger equation  $\Delta_A u + qu = 0$  on  $\Omega$ , where  $\Delta_A$  denotes the magnetic Laplacian defined by  $\Delta_A = \Delta + 2iA(x) \cdot \nabla + i \operatorname{div}_x(A) - |A|^2$ , from some data equivalent to observations of solutions on some parts of the boundary  $\partial\Omega$ .

Yavar Kian, University of Aix-Marseille

### MS4

#### **Lavrentiev-Finite Element Methods for Data Completion Problems with Domain Extensions**

The variational finite element solution of Cauchy's problem, expressed in the Steklov-Poincaré framework and regularized by the Lavrentiev method, has been introduced and computationally assessed in (Azaïez et al., 2011). The present work concentrates on the numerical analysis of the semi-discrete problem. We expose the mathematical study of the error to rigorously establish the convergence of the global bias-variance error. We will speak of the effect of the domain extension on the regularization properties of the discrete problem.

Faker Ben Belgacem, University of Technology Compiègne

V. Girault, F. Jelassi, Université Paris Sorbonne

### MS4

#### **Study of inverse problems by optimization methods**

In this work we consider the inverse problem of obstacle detection for Laplace's equation with partial Cauchy data. The strategy used is to re-



duce the inverse problem into the minimization of a cost-type functional: the Kohn-Vogelius functional. In order to define this functional, we first have to complete the boundary conditions. Hence we begin by focusing on recovering these boundary data, i.e. on the data completion problem, which is also studied through the minimisation of a Kohn-Vogelius functional. Thus the variables of the functional are the shape of the inclusion but also the Cauchy data on the inaccessible part. Due to the ill-posedness of the problem, we regularize the functional through a Tikhonov regularization. After proving some convergence properties for the Cauchy problem, we present several numerical reconstructions of the solution and of the obstacle, from partial boundary data.

Fabian Caubet, Toulouse University

Jérémi Dardé, University Paul Sabatier

Matías Godoy, University of Chile

#### MS4

##### **Steklov-Poincaré algorithm for the Cauchy problem : regularization by spectral analysis**

We propose a study of primal and dual Steklov-Poincaré approaches for the data completion problem in linear elasticity. After giving elementary properties of the discretized operators, we investigate the numerical solution with Krylov solvers. Different preconditioning and acceleration strategies are evaluated. We show that costless filtering of the solution is possible by post-processing Ritz elements. Assessments are provided on a 3D mechanical problem.

Kadri Mohamed Larbi, LAMSIN, University of Tunis El Manar

R. Ferrier, P. Gosselet, ENT Paris-Saclay

#### MS4

##### **Shape Sensitivity Analysis in Linear Elasticity**

This work is devoted to the study of the void

identification problem from partially overdetermined boundary data in the 2D elastostatic case. In a first part, a shape identifiability result from a sub-Cauchy data is presented, i.e. with traction field and tangential component of displacement as measurements. Then, this geometric inverse problem is tackled by the minimization of two cost functionals, an energy gap functional and an L2-gap functional, which enable the reconstruction of voids under Navier's boundary conditions. The shape derivatives of these cost functionals are computed for the purpose of sensitivity analysis.

Bochra Mejri, University of Tunis El Manar

#### MS5

##### **A 3D multiscale modelling framework to investigate the effects of acquired mutations in the EGFR signaling on tumor progression**

The EGFR/ERK signaling cascade is one of the main MAPK pathways that regulate the survival, proliferation, and differentiation of mammalian cells. It is also one of the most altered transduction pathways in cancer. Some of the acquired mutations in the EGFR/ERK pathway can cause the constitutive activation of EGFRs or their overexpression on the surface of the cell. To investigate the role of these alterations in driving tumor progression, we develop a modelling framework of 3D tumor growth integrating the effects of acquired mutations across several scales and capturing the microphysiology of individual cells. In this framework, the interaction of EGFR clusters with their ligands is described with a discrete representation. Cells are depicted as individual objects that move, interact, divide, proliferate, and die by apoptosis. Intracellular regulation dynamics are described using coarse grained-molecular dynamics (MD) in order to preserve the stochastic and spatial aspects of the process. The fate of each cell depends on expression of transcription factors in the nucleus.

We use single-cell simulations to investigate the effects of individual and combined mutations on the regulation of cells. We show that the distribution of active receptor clusters and the distance between them can affect the regulation of individual cells. Then, we describe the role of cancerous mutations in initiating and driving cancer. Finally, we demonstrate the strength and usefulness of the framework by quantifying the impact of the number of EGFR clusters on the growth rate of in silico tumors.

Anass Bouchnita, Uppsala University

Stefan Hellander, Andreas Hellander, Uppsala University

## MS5

### **Simulation of blood flow in a stenosed artery and fractional flow reserve computation**

The objective of this work is to simulate blood flow through a sclerotic artery, using a non-newtonian evolutionary flow model, and a fluid structure interaction model, in order to give a realistic estimation of the virtual FFR : VFFR. The fractional flow reserve FFR measurement has been increasingly used in cardiac catheterization. It provides a quantitative assessment of the functional severity of a coronary artery stenosis identified during coronary angiography and cardiac catheterization. However, this technique remains quite restrictive from a surgical point of view, as long as the value of the distal coronal pressure is obtained by means of a pressure sensor. A realistic simulation of blood flow through a sclerotic wall is necessary to take advantage of this measure, while avoiding surgical complications.

Keltoum Chahour, Abderrahmane Habbal, University of Nice Sophia-Antipolis

Rajae Aboulaich, Ecole Mohammadia of Rabat

Nejib Zemzemi, INRIA Bordeaux

C.Abdelkhirane, Centre Cardiologie Maârif, Casablanca

## MS5

### **Cell plasticity and drug resistance in cancer, with perspectives in optimal therapeutic control**

Drug-induced drug resistance in cancer, the biological and medical question we are tackling from a theoretical point of view, may be due to biological mechanisms of different natures, mere local regulation, epigenetic modifications (reversible, nevertheless heritable) or genetic mutations (irreversible), according to the extent to which the genome of the cells in the population is affected. Reversible or irreversible, drug-induced drug resistance relies on the high phenotypic plasticity of cancer cell populations. In this respect, the modelling framework of adaptive dynamics we will present is likely to biologically correspond to epigenetic modifications, although eventual induction of emergent resistant cell clones due to mutations under drug pressure is not to be excluded. From the biologist's point of view, we study phenotypically heterogeneous, but genetically homogeneous, cancer cell populations under stress by drugs. The built-in targets for theoretical therapeutic control present in the phenotype-structured PDE models we advocate are not supposed to represent well-defined molecular effects of the drugs in use, but rather functional effects, i.e., related to cell death (cytotoxic drugs), or to proliferation in the sense of slowing down the cell division cycle without killing cells (cytostatic drugs). We propose that cell life-threatening drugs (cytotoxics) induce by far more resistance in the highly plastic cancer cell populations than drugs that only limit their growth (cytostatics), and that a rational combination of the two classes of drugs - and possibly others, adding relevant targets to the model - may be optimised to propose therapeutic control strategies to avoid the emergence of drug resistance in tumours. We address this optimal control problem in the context of two

populations, healthy and cancer, both endowed with phenotypes evolving under drug pressure acting as an environmental constraint, and reciprocally inhibiting the proliferation of the other population in a non-local Lotka-Volterra model. We thus have the objective to minimise the proliferation of a cancer cell population while limiting the emergence of drug resistance in it, and taking into account the constraint of limiting toxicity to a population of healthy cells, that are also targets of unwanted adverse effects of the cytotoxic drug. Jean Clairambault, INRIA and LJLL, Sorbonne Université, Luis Almeida, Rebecca Chisholm, Tommaso Lorenzi, Alexander Lorz, Camille Pouchol, Emmanuel Trélat

#### **MS5**

**Helfrich and differential area modeling of vesicles** Vesicles are closed mechanical two dimensional structures made of phospholipids that self-assemble in an aqueous environment. They are the basic mechanical structure of every mammal cells, notably red blood cells. In the seventies, Canham and Helfrich proposed to model those structures as elastic shells whose energy only depends on the mean and Gaussian curvatures. This approach has drawn a lot of attention: Several justifications have been proposed as well as numerical methods. The initial model has also been enriched, what some authors claim to be necessary to fit some experimental observations. Interestingly, it seems that there is no genuine consensus on what the "correct" modeling is In this presentation, we propose to model the structure as a three dimensional structure. Letting the thickness of the vesicle go to zero, we derive several models of vesicles. Finally, we show that the three-dimensional initial modeling can be used to perform numerical simulations. We intend in a nearby future to compare our in silico experiments to actual observations, in order to deter-

mine the more accurate model.

Olivier Pantz, LJAD, University of Nice Sophia-Antipolis

#### **MS6**

##### **Diffusive phenomenon for the wave equation with damping effective at infinity**

We discuss the asymptotic behavior of the damped wave equation in unbounded domains in situations where the damping is effective at infinity. More precisely, we are interested in the rate of decay of the local energy. The energy decay for the contribution of high frequencies is rather well understood, and the uniformity relies on the Geometric Control Condition. In this talk we will focus on the contribution of low frequencies. We will see that in this kind of settings it behaves like the solution of some heat equation. We will illustrate this phenomenon with results in periodic setting and in wave guides.

Julien Royer, Paul Sabatier University

#### **MS6**

##### **Stabilization of coupled wave equation under geometric conditions**

The purpose of this note is to investigate the stabilization of system of two wave equations coupled by velocities with only one localized damping. We expand the results obtained by Alabau et al. (ESAIM 2017) in the case that the damping term is linear, the main novelty of this paper is that the waves are not assumed to be propagate with equal speeds and the coupling coefficient is not assumed to be positive and small. We distinguish two situations. The first one is when the waves propagate at the same speed. In this case, under geometric conditions on the coupling and the damping regions, we establish an exponential energy decay estimate for usual initial data. On the contrary, we first show that our system is not uniformly stable. Next, under the same geomet-

ric conditions, we establish a polynomial energy decay in  $1/t$  for smooth initial data. Finally, in one space dimension, using the real part of the asymptotic expansion of eigenvalues of the system, we prove that the obtained polynomial decay estimate is optimal.

Ali Wehbe, Lebanese University

### MS6

#### **Stabilization of multidimensional wave equation with locally boundary fractional dissipation law under geometric conditions**

In this talk, we consider a multidimensional wave equation with boundary fractional damping acting on a part of the boundary of the domain. First, combining a general criteria of Arendt and Batty with Holmgrens theorem we show the strong stability of our system in the absence of the compactness of the resolvent and without any additional geometric conditions. Next, we show that our system is not uniformly stable in general, since it is the case of the interval. Hence, we look for a polynomial decay rate for smooth initial data for our system by applying a frequency domain approach combining with a multiplier method. Indeed, by assuming that the boundary control region satisfy some geometric conditions and by using the exponential decay of the wave equation with a standard damping, we establish a polynomial energy decay rate for smooth solutions, which depends on the order of the fractional derivative.

Mohammad Akil, Lebanese University

### MS7

#### **Detection of inclusions while recovering boundary data in Stokes Flows using Nash strategies**

We consider the geometric inverse problem where one or more inclusions are to be detected in a Stokes flow, using boundary measurements. In

our case, the boundary data are not available everywhere, but are over-specified in an accessible part of the boundary, while they are missing on the remaining -inaccessible- part. The inverse problem is then of Cauchy type, a family of problems known to be severely ill-posed (in the sense of Hadamard) even without obstacle detection. In order to solve the joint completion/detection problem, we reformulate it as a three players Nash game. The two first players aim at identifying the Dirichlet and Neumann missing data, while the third player aims at identifying the shape(s) of the obstacle(s). We shall present in this talk the framework under which the Nash game is set. We prove the ability of Nash equilibria to capture the missing data for the completion problem, and propose a new algorithm dedicated to the joint computation of the missing data and the obstacle shapes. A level set approach is used for the latter geometric identification problem. Several numerical experiments corroborate the efficiency of our approach.

Abderrahmane Habbal, University of Nice Sophia-Antipolis

### MS7

#### **Optimization of current carrying multi cables using topological and shape gradient strategy**

In this work, we use the topological and shape gradient framework, to optimize a current carrying multi cables. The geometry of the multi cables is modeled as a coated inclusions with different conductivities and the problem we are interested is the location of the inclusions to get a suitable thermal environment. We solve numerically the optimization problem using a gradient-based method and we present some numerical experiments.

Meftahi Belhassen, Houcine Meftahi, LAMSIN, University of Tunis El Manar  
Zakaria Belhachmi, LMIA Haute Alsace b

## MS7

### **Image restoration based on the $p(x)$ -biharmonic operator**

Image restoration plays an important role in various fields and aims to obtain a regularized versions of a noisy or corrupted image. It is a very challenging task mainly in medical imaging and in particular in ultrasound images which are usually highly corrupted with multiplicative noise which affects image analysis methods by making important features hard to detect. In this work, we propose a nonstandard high-order variational model for ultrasound image denoising and we prove its well-posedness. The model is solved via a system of Euler Lagrange equations which give rise to a nonlinear  $p(x)$ -biharmonic PDE. We use an iterative scheme based on the semi-implicit method in order to linearise the obtained fully nonlinear PDE. Then, in order to preserve the important features of the image, we consider a variable exponent function  $p$  chosen adaptively based on the map furnished by edge-detector which is constructed from high-order derivatives. Finally, some numerical results and comparison with other models are presented in order to illustrate the effectiveness of our approach.

Hamdi Houichet, M. Moakher, B. Rjaibi, LAM-SIN, University of Tunis El Manar  
A. Theljani, University of Liverpool

## MS7

### **Modeling Fractures as Interfaces: Model Darcy-Stokes**

In this work, we are interested in the modeling of an incompressible monophasic flow in a porous medium with open fracture. We study the coupling of Darcys model for the flow of fluid in a porous domain with the Stokes equation in the fracture and a boundary conditions on the interface between the two regions. This conditions represent the continuity of mass flux, the continu-

ity of normal stress and Beavers-Joseph-Saffman condition on the tangential stress. The model we developed replaces the fracture with an active interface between the porous rock. His implementation, by using mixed finite element method, was done by the domain decomposition technique. Finally, we present a numerical results verify the convergence of our model for small width fractures.

Kawther Dhifaoui, Ecole Nationale d'Ingenieurs de Tunis

Ali Saada, Ecole Supérieure des Communications de Tunis

## MS8

### **Inverse ECG problem via the factorization method of boundary value problems.**

Electrocardiographic Imaging (ECGI) is a new imaging technique that noninvasively images cardiac electrical activity on the heart surface. In ECGI, a multi-electrode vest records bodysurface potential maps (BSPMs); then, using geometrical information from CT-scans and a mathematical algorithm, electrical potentials, electrograms and isochrones are reconstructed on the heart surface. The reconstruction of cardiac activity from BSPMs is an ill-posed inverse problem. In this work, we present an approach based on an invariant embedding method: the factorization method of boundary values problems. The idea is to embed the initial problem into a family of similar problems on subdomains bounded by a moving boundary from the torso skin to the epicardium surface. For the direct problem this method provides an equivalent formulation with two Cauchy problems evolving on this moving boundary and which have to be solved successively in opposite directions. This method calculates Neumann-Dirichlet and Dirichlet-Neumann operators on the moving boundary using Riccati equations. Mathematical analysis allows to write an optimal

estimation of the epicardial potential based on a quadratic criterion. The output of the method depends on the initial formulation of the problem. Numerical experiments will be presented comparing various approaches.

Jacques Henry, INRIA Bordeaux Sud-Ouest

## MS8

### A game approach to solve the coupled problem of conductivity identification and data completion in cardiac electrophysiology

In this work, we are interested in solving the electrocardiography inverse problem which could be reduced to the data completion problem for the diffusion equation. The difficulty comes from the fact the conductivity values of the torso organs like lungs, bones, liver,...etc, are not known and could be patient dependent. Our goal is to construct a methodology allowing to solve both data completion and conductivity optimization problems at the same time.

We consider the following elliptic problem, where  $\Omega$  is a bounded open domain in  $\mathbb{R}^d$  ( $d = 2, 3$ ) with a sufficiently smooth boundary  $\partial\Omega$  composed of two connected disjoint components  $\Gamma_c$  and  $\Gamma_i$ .

$$\begin{cases} \nabla \cdot (k \nabla u) = 0 & \text{in } \Omega \\ u = f & \text{on } \Gamma_c \\ k \nabla u \cdot \nu = \Phi & \text{on } \Gamma_c \end{cases} \quad (1)$$

The functions  $f$  and  $\Phi$  are the Cauchy data and  $\nu$  is the unit outward normal vector on the boundary and  $k$  is a piecewise constant function representing the unknown conductivity. Data completion problem is already discussed in literature by several methods, as well the identification of parameters has been investigated in many studies. In this work, our purpose is to introduce an original method based on a game theory approach, to write a new algorithm for the identification of conductivity coefficient coupled with a data completion process. We shall say

that there are three players and we define three costs. Each player controls one variable and minimize his own cost in order to seek a Nash equilibrium which is exactly the inverse problem solution. The first player solves the elliptic equation ( $\nabla \cdot (k \nabla(u)) = 0$ ) with the Dirichlet part of the Cauchy data prescribed over the accessible boundary and a variable Neumann condition (which we call first player's strategy) prescribed over the inaccessible part of the boundary. The second player makes uses correspondingly of the Neumann part of the Cauchy data, with a variable Dirichlet condition prescribed over the inaccessible part of the boundary. The first player then minimize the gap related to the nonused Neumann part of the Cauchy data, and so does the second player with a corresponding Dirichlet gap. The two players consider a response of the unknown conductivity of the third player. The third player solves two elliptic problems, depends of the conductivity coefficient (which is the third player's strategy), and uses the over and under specified conditions in the boundary. He minimize then a Kohn-Vogelius type functional. Some numerical 2D and 3D experiments are performed to illustrate the efficiency and robustness of our algorithm. We consider a section of the thorax as a domain composed of three layers (fat, cavity and lung). Each one is characterized by a conductivity. The outer boundary of the thorax plays the role of  $\Gamma_c$ , where the Cauchy data is overspecified. The heart's border plays the role of  $\Gamma_i$ , where the potentiel is missing. We suppose that we have an approximate value of the lung's conductivity (which is equal to 0.5) and we try to improve it. Rabeb Chamekh, Moez Kallel, LAMSIN, University of Tunis El Manar  
Abderrahmane Habbal, University of Nice Sophia-Antipolis  
Nejib Zemzemi, INRIA Bordeaux

## MS8

### **On the identification of multiple space dependent ionic parameters using the bidomain system in cardiac electrophysiology modeling**

The electric wave propagation in the heart can be represented by a non-linear reaction-diffusion system coupled to an ordinary differential equation system called the bidomain model. It takes into account the electrical potential both in the intra-cellular and extra-cellular domains. The coupled system describes the evolution of the transmembrane and the extracellular potentials in the heart. This mathematical model can be formulated as a three-field system (ionic state, transmembrane and extracellular potentials) coupling a non-linear reaction-diffusion equation to an elliptic equation and a non-linear system of ODEs. This model is the so-called bidomain system. In this work, we consider the inverse problem of space dependent multiple ionic parameters identification in cardiac electrophysiology modelling from a set of observations. We use the bidomain system and we consider a general Hodgkin-Huxley formalism to describe the ionic exchanges at the microscopic level. This formalism covers many physiological transmembrane potential models including those in cardiac electrophysiology. Our main result is the proof of the uniqueness and a Lipschitz stability estimate of ion channels conductance parameters based on some observations on an arbitrary subboundary. The key idea is a Carleman estimate for a parabolic operator and elliptic operator with multiple coefficients and an ordinary differential equation system.

Moncef Mahjoub, LAMSIN, University of Tunis El Manar

## MS8

### **Homogenization theory for the derivation of diffusion models**

Diffusion Magnetic Resonance Imaging (dMRI) is an effective tool to obtain useful information on the microscopic structure and has been extensively applied to biological tissues, notably, the brain. We establish a new macroscopic model from homogenization theory for the complex transverse magnetization in a voxel due to diffusion-encoding magnetic gradient in the case of biological tissue with impermeable membranes. In this model, new higher-order diffusion tensors emerge and offer more information about the structure of the biological tissues. We explicitly solve the macroscopic model to obtain an ODE model for the dMRI signal. We have found that the signal based on our model has the same expression of the diffusional kurtosis imaging signal. We also present numerical results for the comparison of our signal with an effective signal.

Marwa Kchaw, LAMSIN, University of Tunis El Manar

## 8 Abstracts of Contributed Talks

### Optimal Control with Exit Time

This talk deals with an optimal control problem with exit time, where we have an initial distribution of agents, given by a measure  $\rho_0$ , in a domain  $\Omega$  and, the goal of each agent is to leave  $\Omega$  through its boundary in minimal time. Each agent is free to move in all directions, but, in order to take into account congestion phenomena, we assume that the maximal speed of each agent is controlled by the following dynamic  $\dot{\gamma}(t) = k(\rho_t, \gamma(t))u(t)$ , where  $\gamma(t)$  gives the position of the agent,  $u(t) \in B(0, 1)$  represents its control,  $\rho_t$  is the evolution of the density of agents at time  $t$ , and  $k$  gives the maximal speed of each agent. First, we will show, briefly, the existence of an equilibrium for such a problem. We will then give a characterization of the equilibrium showing that the evolution density  $\rho_t$  satisfies a continuity equation, whose velocity field depends on the minimal time function  $\varphi$  corresponding to our optimal control problem, coupled with a HamiltonJacobi equation on the value function  $\varphi$ . More precisely, we will show that  $(\rho, \varphi)$  solves

$$\begin{cases} \partial_t \rho(t, x) - \operatorname{div} \left[ \rho(t, x) k(\rho_t, x) \frac{\nabla \varphi(t, x)}{|\nabla \varphi(t, x)|} \right] = 0, \\ \quad (t, x) \in \mathbb{R}_+^* \times \Omega, \\ -\partial_t \varphi(t, x) + |\nabla \varphi(t, x)| k(\rho_t, x) - 1 = 0, \\ \rho(0, \cdot) = \rho_0, \quad (t, x) \in \mathbb{R}_+ \times \Omega, \\ \varphi(t, x) = 0, \quad (t, x) \in \mathbb{R}_+ \times \partial\Omega, \end{cases}$$

The big part of the talk will be dedicated to studying the regularity of the evolution density  $\rho_t$ , at each time  $t \geq 0$ . In particular, we will show that the evolution density  $\rho_t$  belongs to  $L^p$  as soon as the initial density  $\rho_0 \in L^p$ . And to do that, the key idea will be to study the semi-concavity of the value function  $\varphi$ .

Samer Dweik, Paris-Sud University

### A Regularized D-Bar Algorithm for the 2D Real Inverse Conductivity Problem

In this talk, we present a new regularization technique for the Nachman's D-bar method which solves the non linear inverse conductivity problem in dimension two. More particularly, it is the problem of the recovery of a real isotropic conductivity from the boundary data which is represented by the Dirichlet-to-Neumann map. The D-bar method is divided into two steps: firstly recovering the scattering transform from the Dirichlet to Neumann map, then recovering the conductivity from the scattering data. Since the inverse conductivity problem is ill posed, it is important to introduce a regularization which is based on an iterative process starting from the behavior of the CGO solutions. We will present some high order error estimates, then show the stability of the algorithm.

Toufic El Arwadi, Beirut Arab University,  
Sultan El Kontar, Beirut Arab University,  
Toni Sayah, Saint Joseph University

### Stochastic homogenization of a front propagation problem with unbounded velocity

We study the homogenization of Hamilton-Jacobi equations which arise in front propagation problems in stationary ergodic media. Our results are obtained for fronts moving with possible unbounded velocity. We show, by an example, that the homogenized Hamiltonian, which always exists, may be unbounded. In this context, we show convergence results if we start with a compact initial front. On the other hand, if the media satisfies a finite range of dependence condition, we prove that the effective Hamiltonian is bounded and obtain classical homogenization in this context.

Ahmed Hajej, University of Pantheon-Assas